Association Pattern Mining

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Outline

- **Introduction**
- The Frequent Pattern Mining Model
- Association Rule Generation Framework
- Frequent Itemset Mining Algorithms
- Alternative Models: Interesting Patterns
- Useful Meta-algorithms
- Summary

Introduction

- Transactions
  - Sets of items bought by customers
- The Goal
  - Determine associations between groups of items bought by customers
- Quantification of the Level of Association
  - Frequencies of sets of items
- The Discovered Sets of Items
  - Large itemsets, frequent itemsets, or frequent patterns

Applications

- Supermarket Data
  - Target marketing, shelf placement
- Text Mining
  - Identifying co-occurring terms
- Generalization to Dependency-oriented Data Types
  - Web log analysis, software bug detection
- Other Major Data Mining Problems
  - Clustering, classification, and outlier analysis

Association Rules

- Generated from Frequent Itemsets
- Formulation $X \Rightarrow Y$
  - {Beer} ⇒ {Diapers}
  - {Eggs,Milk} ⇒ {Yogurt}
- Applications
  - Promotion
  - Shelf placement
- Conditional Probability

$$P(Y|X) = \frac{P(X \cap Y)}{P(X)}$$

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The Frequent Pattern Mining Model

- U is a set of d iterms
- $\mathcal{T}$ is a set of n transactions $T_1, \ldots, T_n$
  - $T_i \subseteq U$
- Binary Representation of $T_1, \ldots, T_n$
  - $U = \{Bread, Butter, Cheese, Eggs, Milk, Yogurt\}$

tid	Set of items	Binary representation
1	$\{Bread, Butter, Milk\}$	110010
2	$\{Eggs, Milk, Yogurt\}$	000111
3	$\{Bread, Cheese, Eggs, Milk\}$	101110

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- Itemset, k-itemset
  - A set of items, A set of k items

Definitions

Support

Definition 4.2.1 (Support) *The support of an itemset I is defined as the fraction of the transactions in the database $\mathcal{T} = \{T_1 \dots T_n\}$ that contain I as a subset.*

- Denoted by $sup(I)$

Frequent Itemset Mining

Definition 4.2.2 (Frequent Itemset Mining) *Given a set of transactions $\mathcal{T} = \{T_1 \dots T_n\}$, where each transaction T_i is a subset of items from U, determine all itemsets I that occur as a subset of at least a predefined fraction minsup of the transactions in $\mathcal{T}$.*

- $minsup$ is the minimum support

Definition 4.2.3 (Frequent Itemset Mining: Set-wise Definition) *Given a set of sets $\mathcal{T} = \{T_1 \dots T_n\}$, where each element of the set T_i is drawn on the universe of elements U, determine all sets I that occur as a subset of at least a predefined fraction minsup of the sets in $\mathcal{T}$.*

An Example

- A Market Basket Data Set

tid	Set of items	Binary representation
1	$\{Bread, Butter, Milk\}$	110010
2	$\{Eggs, Milk, Yogurt\}$	000111
3	$\{Bread, Cheese, Eggs, Milk\}$	101110
4	$\{Eggs, Milk, Yogurt\}$	000111
5	$\{Cheese, Milk, Yogurt\}$	001011

  - $support$ of $\{Bread, Milk\}$ is $\frac{2}{5} = 0.4$
  - $support$ of $\{Cheese, Yogurt\}$ is $\frac{1}{5} = 0.2$
- $minsup = 0.3$
  - $\{Bread, Milk\}$ is a frequent itemset

Properties

- The smaller *minsup* is, the larger the number of frequent itemsets is.
- Support Monotonicity Property

Property 4.2.1 (Support Monotonicity Property) *The support of every subset J of I is at least equal to that of the support of itemset I.*

$$sup(J) \geq sup(I) \quad \forall J \subseteq I \tag{4.1}$$

  - When an itemset I is contained in a transaction, all its subsets will also be contained in the transaction.

- Downward Closure Property

Property 4.2.2 (Downward Closure Property) *Every subset of a frequent itemset is also frequent.*

Maximal Frequent Itemsets

Definition 4.2.4 (Maximal Frequent Itemsets) *A frequent itemset is maximal at a given minimum support level minsup, if it is frequent, and no superset of it is frequent.*

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1	$\{Bread, Butter, Milk\}$	110010
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- Maximal frequent patterns at $minsup = 0.3$
  - {Bread,Milk}, {Cheese,Milk}, {Eggs,Milk, Yogurt}
- Frequent Patterns at $minsup = 0.3$
  - The total number is 11
  - Subsets of the maximal frequent patterns

Maximal Frequent Itemsets

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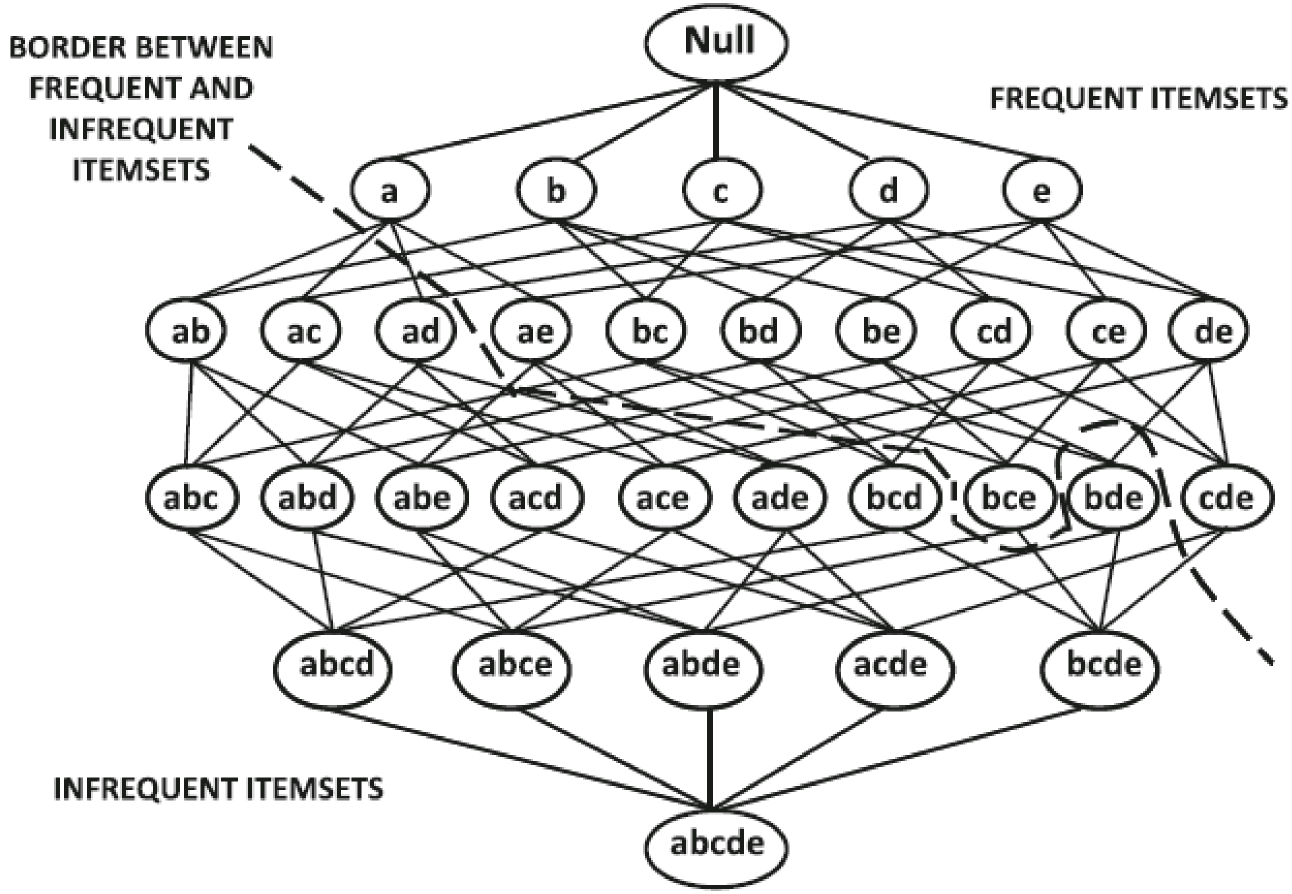
- The maximal patterns can be considered condensed representations of the frequent patterns.
- However, this condensed representation does not retain information about the support values of the subsets.

The Itemset Lattice

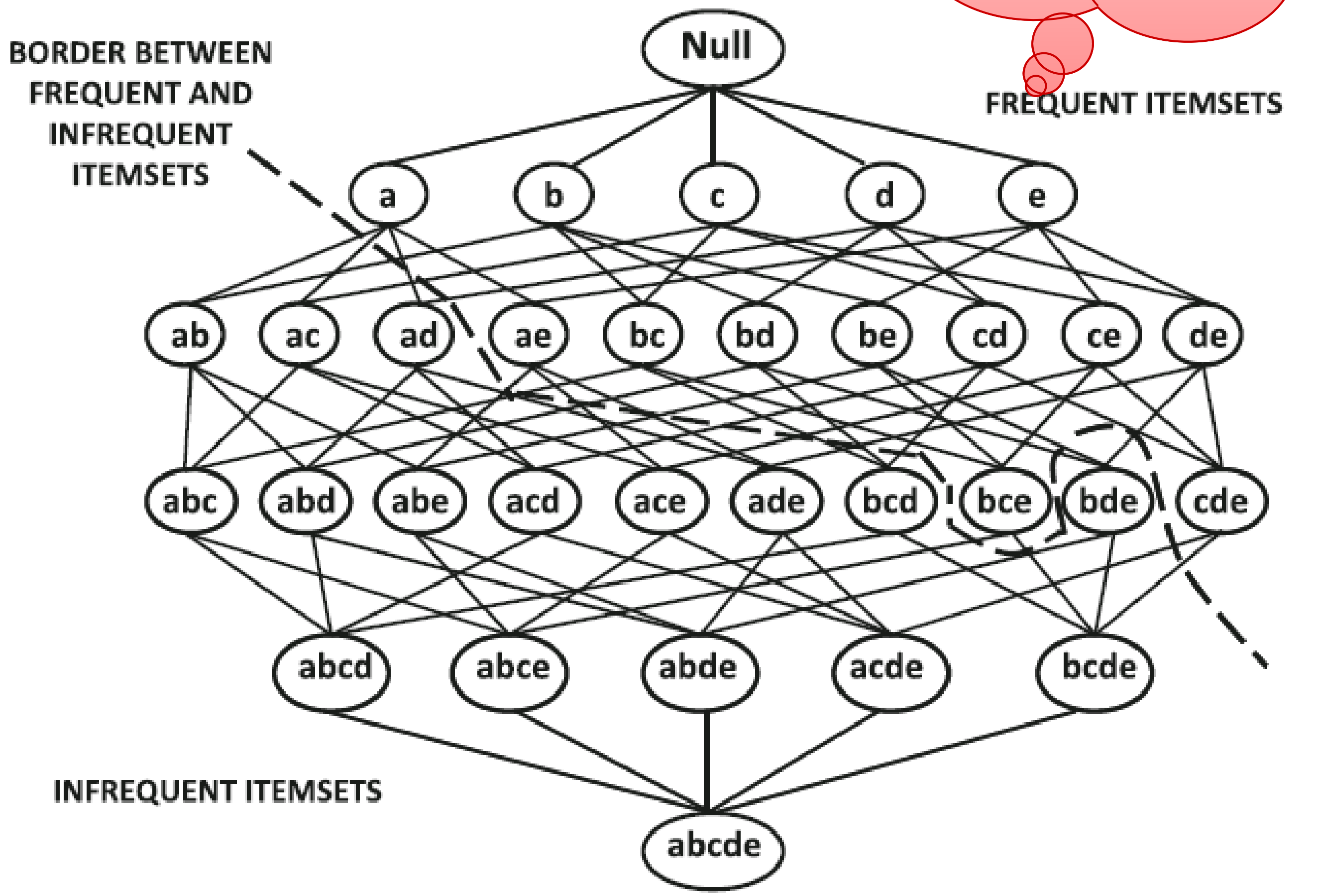
- Contain $2^{|U|}$ nodes and represents search space



The Itemset Lattice

- Contain $2^{|U|}$ nodes and repr

All itemsets above this border are frequent.



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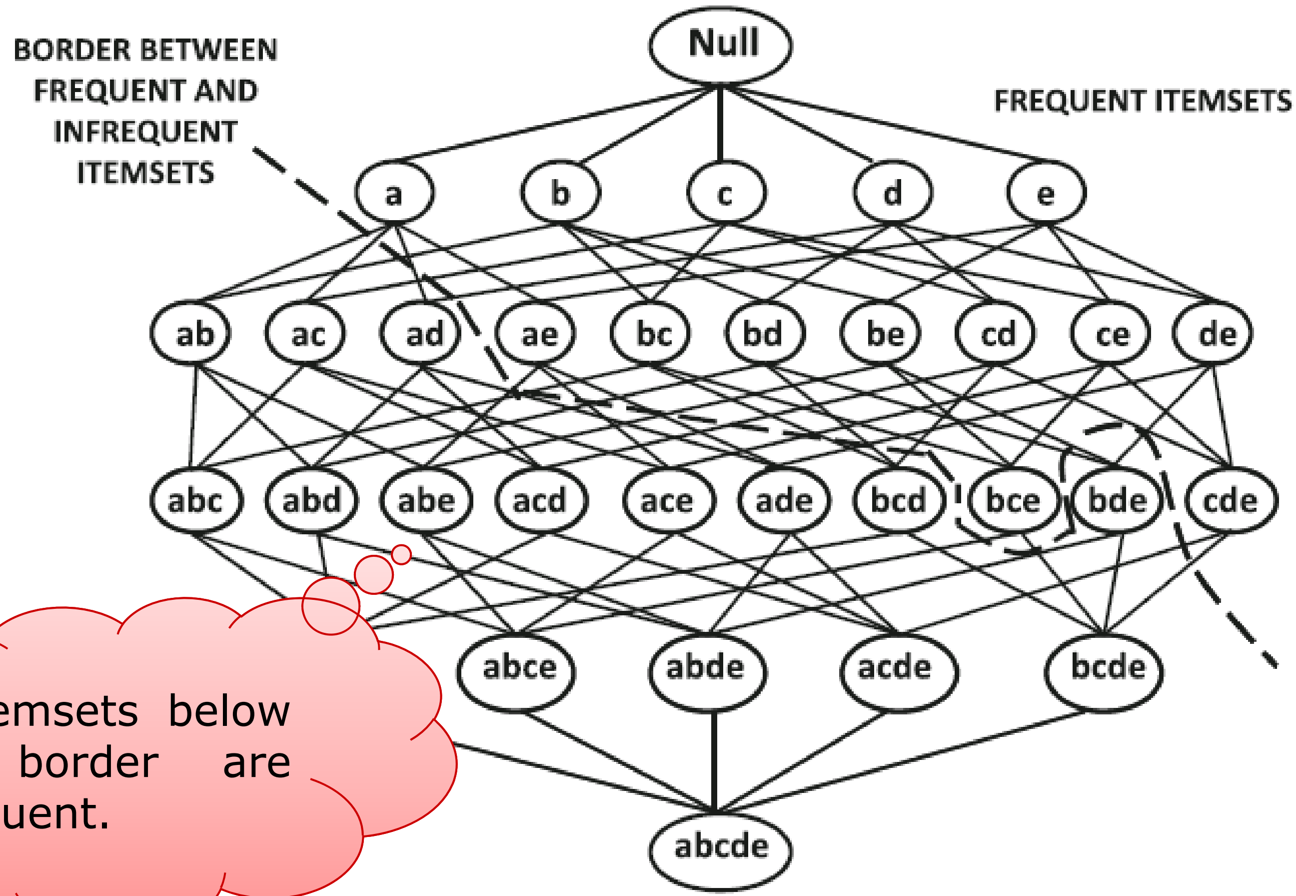

All itemsets below this border are infrequent.

The Itemset Lattice

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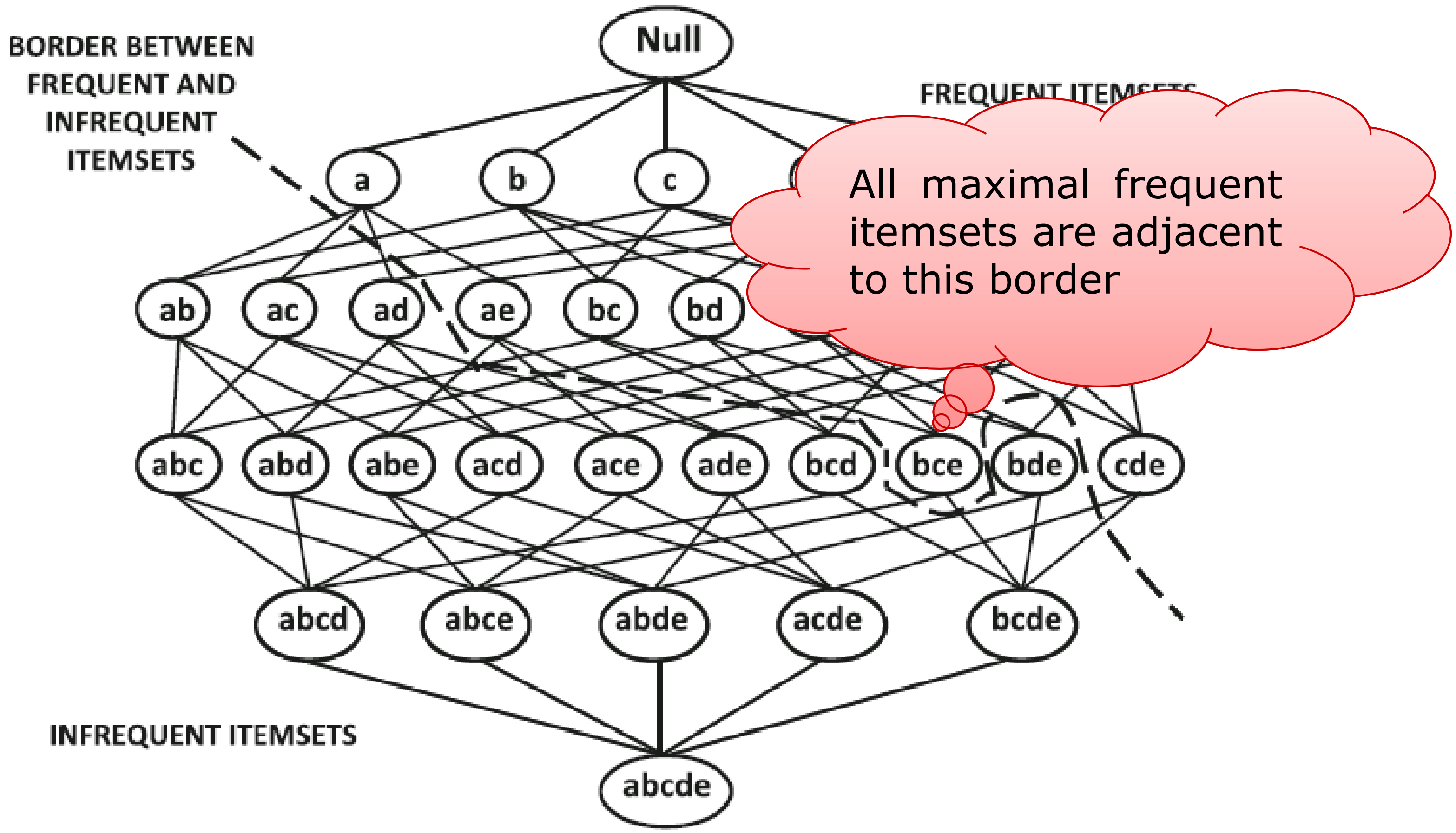

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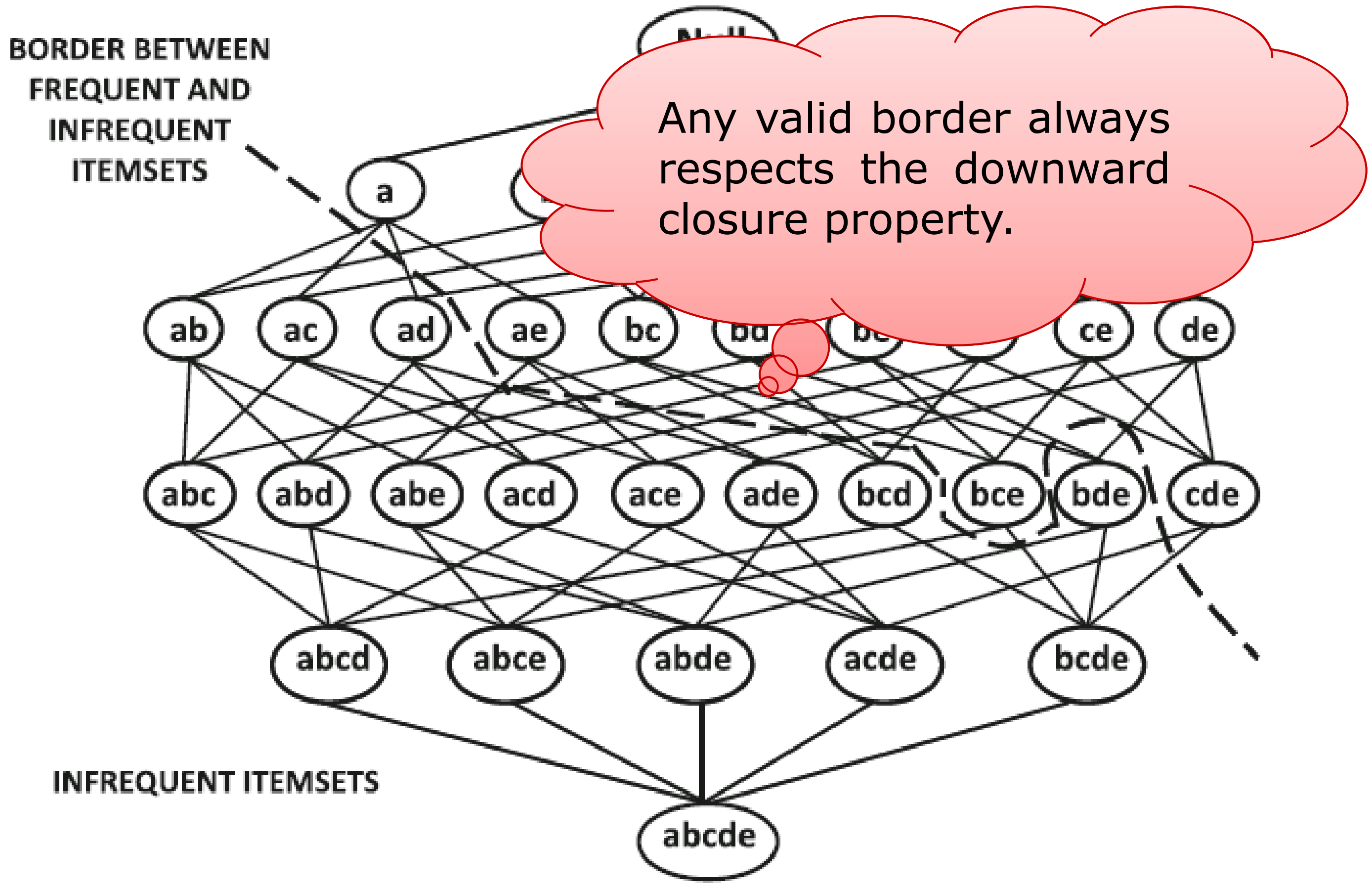

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Definitions

- The confidence of a rule $X \Rightarrow Y$

Definition 4.3.1 (Confidence) *Let X and Y be two sets of items. The confidence $conf(X \Rightarrow Y)$ of the rule $X \Rightarrow Y$ is the conditional probability of $X \cup Y$ occurring in a transaction, given that the transaction contains X. Therefore, the confidence $conf(X \Rightarrow Y)$ is defined as follows:*

$$conf(X \Rightarrow Y) = \frac{sup(X \cup Y)}{sup(X)}. \tag{4.2}$$

- X and Y are said to be the antecedent and the consequent
- In the previous table

$$conf(\{Eggs, Milk\} \Rightarrow \{Yogurt\}) = \frac{sup(\{Eggs, Milk, Yogurt\})}{sup(\{Eggs, Milk\})} = \frac{0.4}{0.6} = \frac{2}{3}$$

Definitions

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Association Rules

Definition 4.3.2 (Association Rules) *Let X and Y be two sets of items. Then, the rule $X \Rightarrow Y$ is said to be an association rule at a minimum support of minsup and minimum confidence of minconf, if it satisfies both the following criteria:*

1. *The support of the itemset $X \cup Y$ is at least minsup.*
2. *The confidence of the rule $X \Rightarrow Y$ is at least minconf.*

- A sufficient number of transactions are relevant
- A sufficient strength in terms of conditional probabilities

The Overall Framework

1. In the first phase, all the frequent itemsets are generated at the minimum support of $minsup$.
   - The most difficult step

2. In the second phase, the association rules are generated from the frequent itemsets at the minimum confidence level of $minconf$.
   - Relatively straightforward

Implementation of 2nd Phase

- A Straightforward Implentation
  - Given a frequent itemset I
  - Generate all possible partitions X and $Y = I - X$
  - Examine the confidence of each $X \Rightarrow Y$
- Reduce the Search Space

Property 4.3.1 (Confidence Monotonicity) *Let X_1, X_2, and I be itemsets such that $X_1 \subset X_2 \subset I$. Then the confidence of $X_2 \Rightarrow I - X_2$ is at least that of $X_1 \Rightarrow I - X_1$.*

$$conf(X_2 \Rightarrow I - X_2) \geq conf(X_1 \Rightarrow I - X_1) \tag{4.3}$$

$$sup(X_2) \leq sup(X_1) \Rightarrow \frac{sup(I)}{sup(X_2)} \geq \frac{sup(I)}{sup(X_1)}$$

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$$conf(X_2 \Rightarrow I - X_2) \geq conf(X_1 \Rightarrow I - X_1) \tag{4.3}$$

  - Techniques for frequent itemsets mining can also be applied here

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Frequent Itemset Mining Algorithms

- **Brute Force Algorithms**

- The Apriori Algorithm
  - The 2nd homework

- Enumeration-Tree Algorithms

- Recursive Suffix-Based Pattern Growth Methods

Brute Force Algorithms (1)

- The Naïve Approach
  - Generate all these candidate itemsets
    - ✓ For a universe of items U, there are a total of $2^{|U|} - 1$ distinct subsets
    - ✓ When $U = 1000$, $2^{1000} \geq 10^{300}$
  - Count their support against the transaction database
- Observation
  - no $(k+1)$-patterns are frequent if no k-patterns are frequent.

Brute Force Algorithms (2)

A Improved Approach

- Generate all candidate k-itemsets with k
- Count their support against the transaction database
- If no frequent itemsets are found, then stop; Otherwise, $k++$ and continue;

A Significant Improvement

- Let l be the final value of k

$$\sum_{i=1}^{l} \binom{|U|}{i} \ll 2^{|U|}$$

- $|U| = 1000$ and $l = 10$, it is $O(10^{23})$

Brute Force Algorithms (3)

- A very minor application of the downward closure property made the algorithm much faster
- To Further Improve the Efficiency

1. Reducing the size of the explored search space (lattice of Fig. 4.1) by pruning candidate *itemsets* (lattice nodes) using tricks, such as the *downward closure* property.
2. Counting the support of each candidate more efficiently by pruning *transactions* that are known to be irrelevant for counting a candidate itemset.
3. Using compact data structures to represent either candidates or transaction databases that support efficient counting.

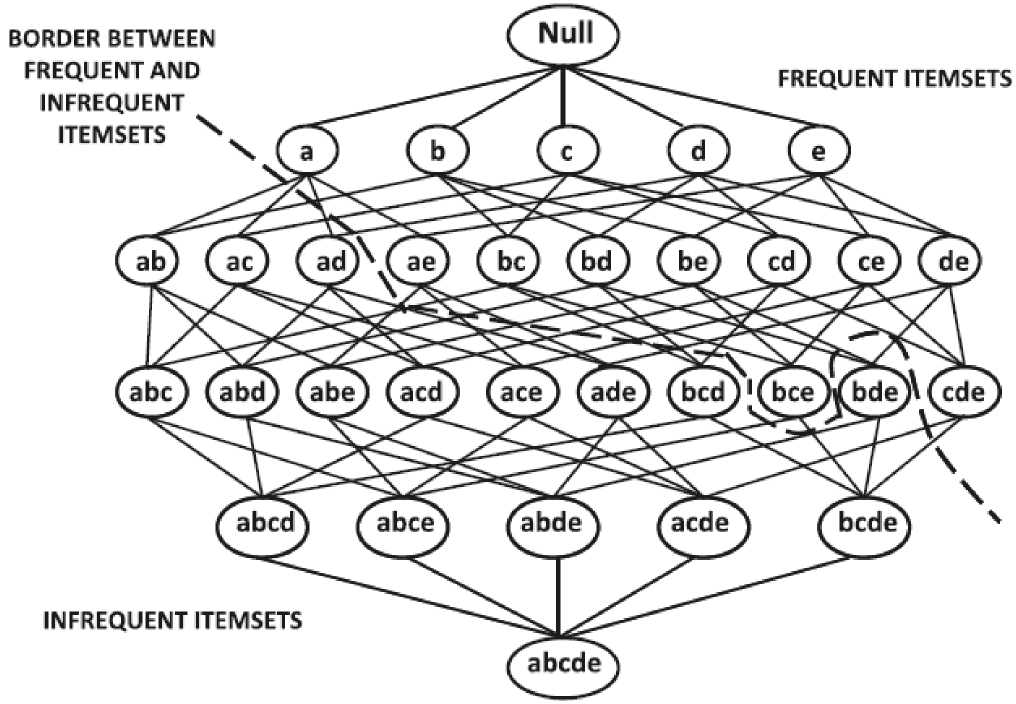
Frequent Itemset Mining Algorithms

- Brute Force Algorithms
- **The Apriori Algorithm**
  - The 2nd homework
- Enumeration-Tree Algorithms
- Recursive Suffix-Based Pattern Growth Methods

The Apriori Algorithm

The Basic Idea

- Use the downward closure property to prune the candidate search space



The Apriori Algorithm

- The Basic Idea
  - Use the downward closure property to prune the candidate search space
- The Overall Procedure (level-wise)
  - Using the frequent k-itemsets to generate $(k+1)$-candidates
  - Prune the candidates before counting
  - Counts the supports of the remaining $(k+1)$-candidates
  - Stop if there is no frequent $(k+1)$-itemsets

The pseudocode

Algorithm *Apriori*(Transactions: $\mathcal{T}$, Minimum Support: $minsup$)
begin
  $k = 1$;
  $\mathcal{F}_1 = \{$ All Frequent 1-itemsets $\}$;
  while $\mathcal{F}_k$ is not empty **do begin**
    Generate $\mathcal{C}_{k+1}$ by joining itemset-pairs in $\mathcal{F}_k$;
    Prune itemsets from $\mathcal{C}_{k+1}$ that violate downward closure;
    Determine $\mathcal{F}_{k+1}$ by support counting on $(\mathcal{C}_{k+1}, \mathcal{T})$ and retaining
        itemsets from $\mathcal{C}_{k+1}$ with support at least $minsup$;
    $k = k + 1$;
  end;
  return$(\cup_{i=1}^{k} \mathcal{F}_i)$;
end

Candidates Generation (1)

- A Naïve Approach
  - Check all the possible combination of frequent k-itemsets
  - Keep all the $(k+1)$-itemsets
- An Example of the Naive Approach
  - k-itemsets: {abc} {bcd} {abd} {cde}
  - {abc} + {bcd} = {abcd}
  - {bcd} + {abd} = {abcd}
  - {abd} + {cde} = {abcde}
  - ……

Candidates Generation (1)

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  - {bcd} + {abd} = {abcd}
  - {abd} + {cde} = {abcde}
  - ……

Redundancy
and
Inefficiency

Candidates Generation (2)

- Introduction of Ordering
  - Items in U have a lexicographic ordering
  - Itemsets can be order as strings

- A Better Approach
  - Order the frequent k-itemsets
  - Merge two itemset if the first $k-1$ items of them are the same

Candidates Generation (3)

- Examples of the New Methods
  - k-itemsets: {abc} {abd} {bcd}
  - {abc} + {abd} = {abcd}

  - k-itemsets: {abc} {acd} {bcd}
  - No $(k+1)$-candidates
  - Early stop is possible
    - ✓ Donot need to check {abc} +{bcd} after checking {abc} + {acd}
  - Do we miss {abcd}?
    - ✓ No, due to the Downward Closure Property

Level-wise Pruning Trick

- Let F_k be the set of frequent k-itemsets
- Let C_{k+1} be the set of $(k+1)$-candidates
- For an $I \in C_{k+1}$, it is frequent only if all the all the k-subsets of I are frequent
- Pruning
  - Generate all the k-subsets of I
  - If any one of them does not belong to F_k, then remove I

Support Counting (1)

- A Naïve Approach
  - For each candidate $I_i \in C_{k+1}$
    - ✓ For each transaction T_j in the transaction database T
      - Check whether I_i appears in T_j

- The Limitation
  - Inefficient if both $|C_{k+1}|$ and $|T|$ are very large

Support Counting (2)

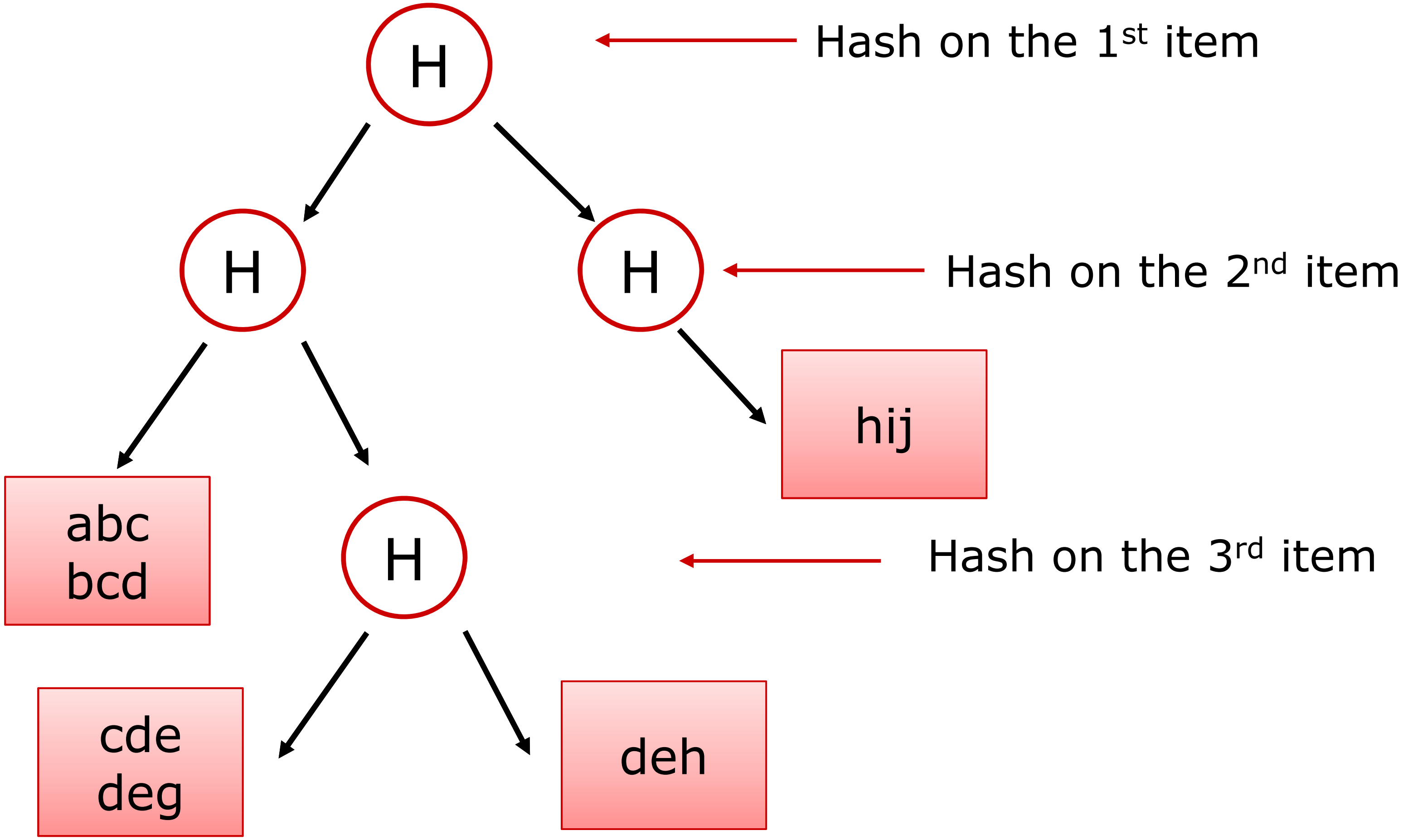
- A Better Apporach
  - Organize the candidate patterns in C_{k+1} with a hash tree
    - ✓ Hash tree construction
  - Use the hash tree to accelerate counting
    - ✓ Each transaction T_i is examined with a small number of candidates in C_{k+1}

Hash Tree

- A tree with a fixed degree of the internal nodes
- Each internal node is associated with a random hash function that maps an item to one of its children
- A leaf node contains a list of lexicographically sorted itemsets
- Every itemset in C_{k+1} is contained in exactly one leaf node of the hash tree.

Hash Tree of C_3

- The Maximum Depth is $3 + 1$



Counting based on Hash Tree

- For each T_j, identify leaves in the hash tree that might contain subset items
- The Procedure
  - Root node – hash on all items in T_j
    - ✓ Suppose the i-th item of T_j is hashed to one node, then pass this position i to that node
  - If we are at a leaf – find all itemsets contained in T_j
  - If we are at an interior node – hash on each item after the given position
    - ✓ Suppose the i-th item of T_j is hashed to one node, then pass this position i to that node

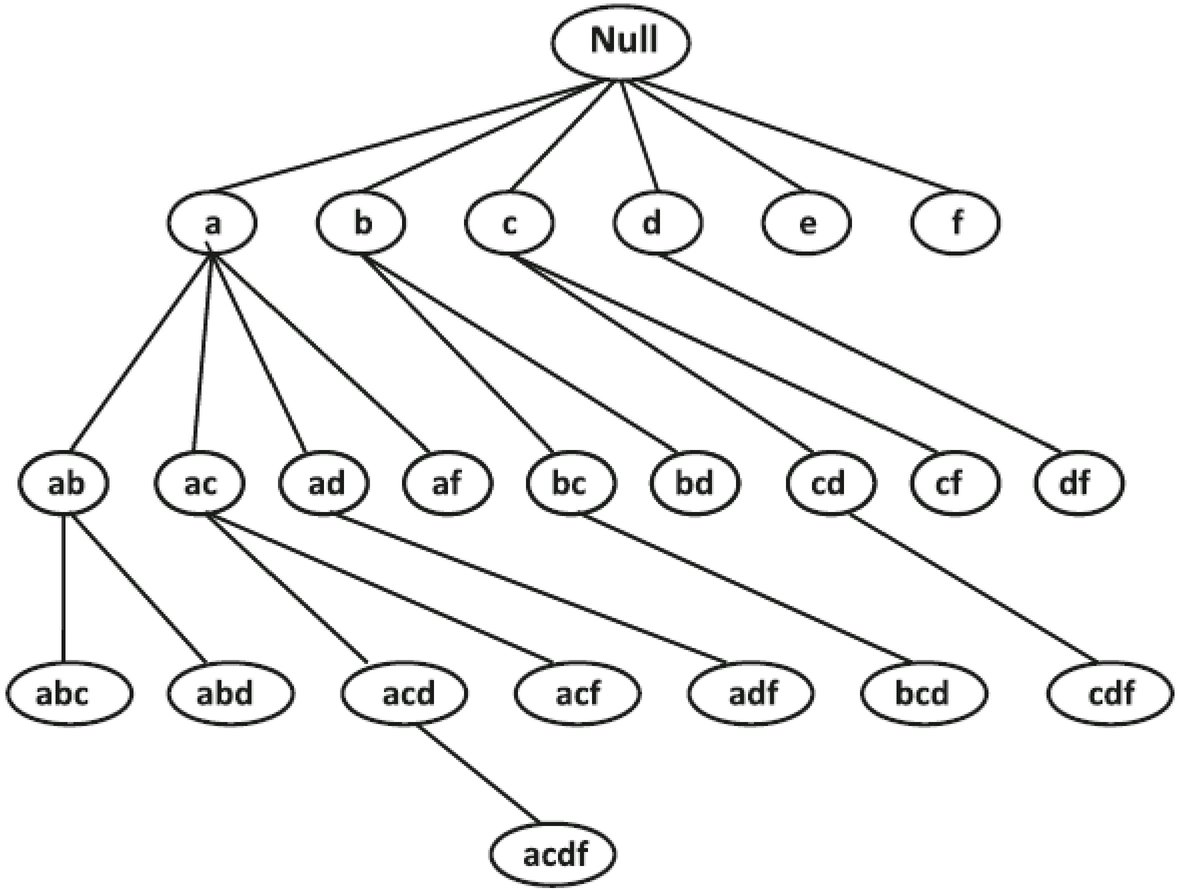
Frequent Itemset Mining Algorithms

- Brute Force Algorithms
- The Apriori Algorithm
  - The 2nd homework
- **Enumeration-Tree Algorithms**
- Recursive Suffix-Based Pattern Growth Methods

Enumeration-Tree

- Lexicographic Tree
  - A node exists in the tree corresponding to each frequent itemset.
  - The root of the tree corresponds to the null itemset.
  - Let $I = \{i_1, \ldots, i_k\}$ be a frequent itemset, where $i_1, \ldots, i_k$ are listed in lexicographic order. The parent of the node I is the itemset $\{i_1, \ldots, i_{k-1}\}$

An Example



- Frequent Tree Extension
  - An item that is used to extend a node

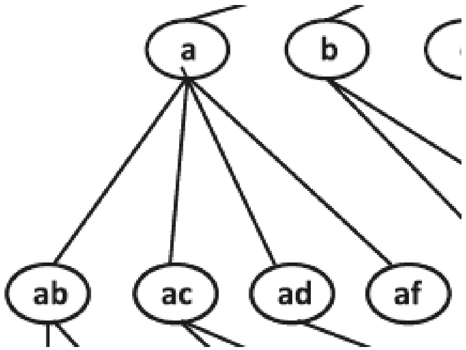
Enumeration Tree Algorithms

Algorithm *GenericEnumerationTree*(Transactions: $\mathcal{T}$,
Minimum Support: *minsup*)
begin
  Initialize enumeration tree $\mathcal{ET}$ to single $Null$ node;
  while any node in $\mathcal{ET}$ has not been examined **do begin**
    Select one of more unexamined nodes $\mathcal{P}$ from $\mathcal{ET}$ for examination;
    Generate candidates extensions $C(P)$ of each node $P \in \mathcal{P}$;
    Determine frequent extensions $F(P) \subseteq C(P)$ for each $P \in \mathcal{P}$ with support counting;
    Extend each node $P \in \mathcal{P}$ in $\mathcal{ET}$ with its frequent extensions in $F(P)$;
  end
  return enumeration tree $\mathcal{ET}$;
end

- Let Q be the parent of P
- Let $F(Q)$ be the frequent extensions of Q
- Then, $C(P) \subseteq F(Q)$

Enumeration-Tree-Based Interpretation of Apriori

- Apriori constructs the enumeration tree in breadth-first manner
- Apriori generates candidate $(k+1)$-itemsets by merging two frequent k-itemsets of which the first $k-1$ items of are the same



- Extend $\{ab\}$ with $\{cdf\} \subseteq F(\{a\})$

TreeProjection (1)

- The Goal
  - Reuse the counting work that has already been done before
- Projected Databases
  - Each projected transaction database is specific to an enumeration-tree node.
  - Transactions that do not contain the itemset P are removed.
  - Projected database at node P can be expressed only in terms of the items in $C(P)$

TreeProjection (2)

The Algorithm

Algorithm *ProjectedEnumerationTree*(Transactions: $\mathcal{T}$,
Minimum Support: $minsup$)
begin
Initialize enumeration tree $\mathcal{ET}$ to a single $(Null, \mathcal{T})$ root node;
while any node in $\mathcal{ET}$ has not been examined **do begin**
Select an unexamined node $(P, \mathcal{T}(P))$ from $\mathcal{ET}$ for examination;
Generate candidates item extensions $C(P)$ of node $(P, \mathcal{T}(P))$;
Determine frequent item extensions $F(P) \subseteq C(P)$ by support counting
of individual items in smaller projected database $\mathcal{T}(P)$;
Remove infrequent items in $\mathcal{T}(P)$;
for each frequent item extension $i \in F(P)$ **do begin**
Generate $\mathcal{T}(P \cup \{i\})$ from $\mathcal{T}(P)$;
Add $(P \cup \{i\}, \mathcal{T}(P \cup \{i\}))$ as child of P in $\mathcal{ET}$;
end
end
return enumeration tree $\mathcal{ET}$;
end

Vertical Counting Methods (1)

- Vertical Representation of Market Basket Data Set

Item	Set of tids	Binary representation
Bread	$\{1,3\}$	10100
Butter	$\{1\}$	10000
Cheese	$\{3,5\}$	00101
Eggs	$\{2,3,4\}$	01110
Milk	$\{1,2,3,4,5\}$	11111
Yogurt	$\{2,4,5\}$	01011

- Intersection of two item *tid* list gives a new list
  - The length is the support of the 2-itemset

Vertical Counting Methods (2)

- The Algorithm

Algorithm *VerticalApriori*(Transactions: $\mathcal{T}$, Minimum Support: *minsup*)
begin
  $k = 1$;
  $\mathcal{F}_1 = \{$ All Frequent 1-itemsets $\}$;
  Construct vertical *tid* lists of each frequent item;
  while $\mathcal{F}_k$ is not empty **do begin**
    Generate $\mathcal{C}_{k+1}$ by joining itemset-pairs in $\mathcal{F}_k$;
    Prune itemsets from $\mathcal{C}_{k+1}$ that violate downward closure;
    Generate *tid* list of each candidate itemset in $\mathcal{C}_{k+1}$ by intersecting
      tid lists of the itemset-pair in $\mathcal{F}_k$ that was used to create it;
    Determine supports of itemsets in $\mathcal{C}_{k+1}$ using lengths of their *tid* lists;
    $\mathcal{F}_{k+1}$= Frequent itemsets of $\mathcal{C}_{k+1}$ together with their *tid* lists;
    $k = k + 1$;
  end;
  return($\cup_{i=1}^{k} \mathcal{F}_i$);
end

Frequent Itemset Mining Algorithms

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- **Recursive Suffix-Based Pattern Growth Methods**

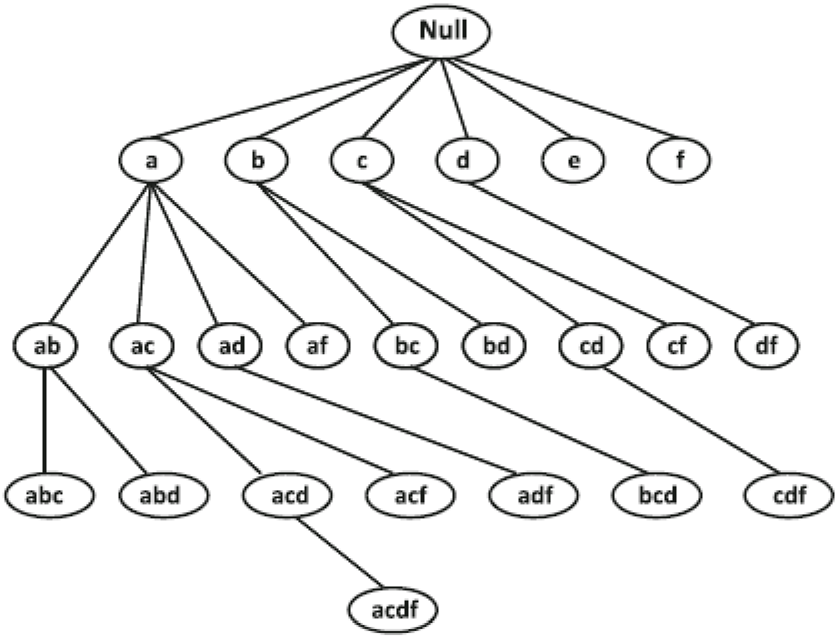
Generic Recursive Suffix Growth Algorithm

- $\mathcal{T}$ is expressed in terms of only frequent 1-itemset

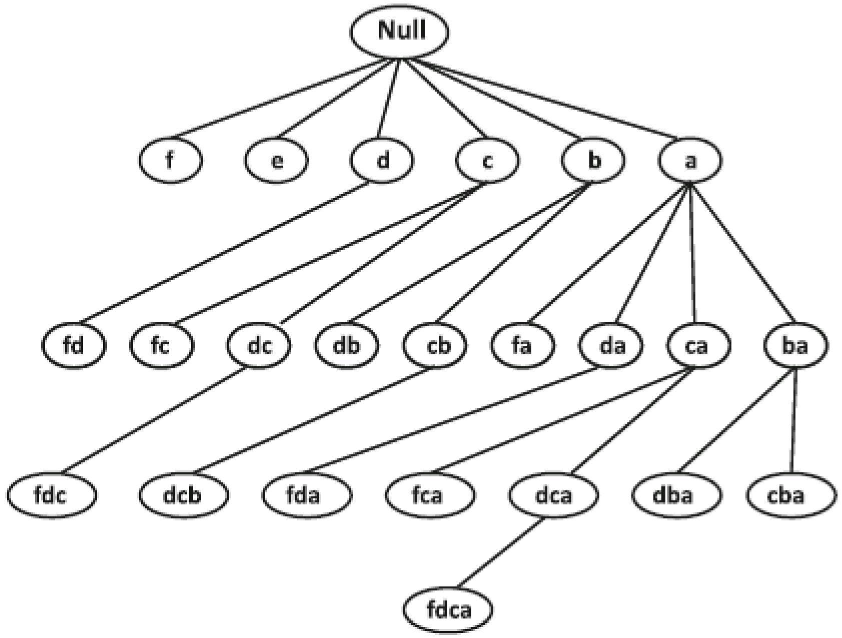
Algorithm *RecursiveSuffixGrowth*(Transactions in terms of frequent 1-items: $\mathcal{T}$,
Minimum Support: *minsup*, Current Suffix: P)
begin
  for each item i in $\mathcal{T}$ **do begin**
    report itemset $P_i = \{i\} \cup P$ as frequent;
    Extract all transactions $\mathcal{T}_i$ from $\mathcal{T}$ containing item i;
    Remove all items from $\mathcal{T}_i$ that are lexicographically $\geq i$;
    Remove all infrequent items from $\mathcal{T}_i$;
    if $(\mathcal{T}_i \neq \phi)$ **then** *RecursiveSuffixGrowth*$(\mathcal{T}_i, minsup, P_i)$;
  end
end

Relationship Between FP-Growth and Enumeration-Tree Methods

- They are Equivalent



(a) Prefix extensions with ordering of a, b, c, d, e, f (Enumeration Tree Prefixes shown)



(b) *FP-growth* with ordering of f, e, d, c, b, a (Recursion Tree Suffixes shown)

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Motivations (1)

- Advantages of Frequent Itemsets
  - Very simple and intuitive
    - ✓ Raw frequency for the support
    - ✓ Conditional probabilities for the confidence
  - Downward Closure Property
    - ✓ Enable efficient algorithms

Motivations (2)

Disadvantages of Frequent Itemsets

- Patterns are not always significant from an application-specific perspective

tid	Set of items	Binary representation
1	$\{Bread, Butter, Milk\}$	110010
2	$\{Eggs, Milk, Yogurt\}$	000111
3	$\{Bread, Cheese, Eggs, Milk\}$	101110
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  - ✓ *Milk* can be appended to any set of items, without changing its frequency
  - ✓ For any set of items X, the association rule $X \Rightarrow \{Milk\}$ has 100% confidence

Motivations (2)

- **Disadvantages of Frequent Itemsets**
  - Patterns are not always significant from an application-specific perspective
  - Cannot adjust to the skew in the individual item support values
    - ✓ Support of $\{Milk, Butter\}$ is very different from $\{\neg Milk, \neg Butter\}$
    - ✓ But the statistical coefficient of correlation is exactly the same in both cases
- **Bit Symmetric Property**
  - Values of 0 in the binary matrix are treated in a similar way to values of 1

Statistical Coefficient of Correlation

- Pearson Coefficient

$$\rho = \frac{E[X \cdot Y] - E[X] \cdot E[Y]}{\sigma(X) \cdot \sigma(Y)}$$

- Estimated Correlation

$$\rho_{ij} = \frac{sup(\{i,j\}) - sup(i) \cdot sup(j)}{\sqrt{sup(i) \cdot sup(j) \cdot (1 - sup(i)) \cdot (1 - sup(j))}}.$$

- Properties
  - Lies in the range $[-1,1]$
  - Satisfies the bit symmetric property
  - Intuitively hard to interpret

χ^2 Measure

- Given a set X of k items, there are 2^k-possible states
  - $k = 2$ items $\{Bread, Butter\}$, the 2^2 states are $\{Bread, Butter\}$, $\{Bread, \neg Butter\}$, $\{\neg Bread, Butter\}$, and $\{\neg Bread, \neg Butter\}$

χ^2 Measure

- Given a set X of k items, there are 2^k-possible states
- The χ^2-measure for set of items X

$$\chi^2(X) = \sum_{i=1}^{2^{|X|}} \frac{(O_i - E_i)^2}{E_i}.$$

  - O_i and E_i be the observed and expected values of the absolute support of state i

χ^2 Measure

- Given a set X of k items, there are 2^k-possible states
- The χ^2-measure for set of items X
- Properties
  - Larger values of this quantity indicate greater dependence
  - Do not reveal whether the dependence between items is positive or negative
  - Is bit-symmetric
  - Satisfies the upward closure property
  - High computational complexity

Interest Ratio

- Definition

$$I(\{i_1 \dots i_k\}) = \frac{sup(\{i_1 \dots i_k\})}{\prod_{j=1}^{k} sup(i_j)}$$

- Properties
  - When the items are statistically independent, the ratio is 1.
  - Value greater than 1 indicates that the variables are positively correlated.
  - When some items are extremely rare, the interest ratio can be misleading.
  - Not satisfy the downward closure property.

Symmetric Confidence Measures

- Confidence Measure is Asymmetric

$$conf(X \Rightarrow Y) \neq conf(Y \Rightarrow X)$$

- Let X and Y be two 1-itemsets
  - Minimum of $conf(X \Rightarrow Y)$ and $conf(Y \Rightarrow X)$
  - Maximum of $conf(X \Rightarrow Y)$ and $conf(Y \Rightarrow X)$
  - Average of $conf(X \Rightarrow Y)$ and $conf(Y \Rightarrow X)$
    - ✓ Geometric mean is the cosine measure
- Can be generalized to k-itemsets
- Do not satisfy the downward closure property

Cosine Coefficient on Columns

- Definition

$$cosine(i,j) = \frac{sup(\{i,j\})}{\sqrt{sup(i)} \cdot \sqrt{sup(j)}}.$$

- Interpretation
  - Cosine similarity between two columns of the data matrix

- A Symmetric Confidence Measure

Jaccard Coefficient and the Min-hash Trick

- Jaccard coefficient $J(S_1, S_2)$ between the two sets

$$J(S_1, S_2) = \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|}$$

- Jaccard coefficient between multiway sets

$$J(S_1 \dots S_k) = \frac{|\cap S_i|}{|\cup S_i|}.$$

- Properties
  - Satisfy the downward closure property
  - Speed up by min-hash trick

Collective Strength (1)

- Violation
  - If some of the items of I are present in the transaction, and others are not.

- Violation Rate $v(I)$
  - The fraction of violations of the itemset I over all transactions.

Collective Strength (2)

Collective Strength

$$C(I) = \frac{1 - v(I)}{1 - E[v(I)]} \cdot \frac{E[v(I)]}{v(I)}.$$

- The expected value of $v(I)$ is calculated assuming statistical independence of the individual items.

$$E[v(I)] = 1 - \prod_{i \in I} p_i - \prod_{i \in I} (1 - p_i).$$

- 0 indicates a perfect negative correlation
- ∞ indicates a perfectly positive correlation

Collective Strength (3)

Interpretation of Collective Strength

$$C(I) = \frac{\text{Good Events}}{\text{E[Good Events]}} \cdot \frac{\text{E[Bad Events]}}{\text{Bad Events}}.$$

Strongly Collective Itemsets

Definition 4.5.1 *An itemset I is denoted to be strongly collective at level s, if it satisfies the following properties:*

1. *The collective strength $C(I)$ of the itemset I is at least s.*
2. **Closure property:** *The collective strength $C(J)$ of every subset J of I is at least s.*

- The closure property is enforced.

Relationship to Negative Pattern Mining

- Motivation
  - Determine patterns between items or their absence
- Satisfy Bit Symmetric Property
  - Statistical coefficient of correlation
  - χ^2 measure
  - Jaccard Coefficient, Strongly Collective strength
    - ✓ Also satisfy downward closure property

Outline

- Introduction
- The Frequent Pattern Mining Model
- Association Rule Generation Framework
- Frequent Itemset Mining Algorithms
- Alternative Models: Interesting Patterns
- **Useful Meta-algorithms**
- Summary

Useful Meta-algorithms

- Definition
  - An algorithm that uses a particular algorithm as a subroutine
    - either to make the original algorithm more efficient (e.g., by sampling)
    - or to gain new insights
- Sampling Methods
- Data Partitioned Ensembles
- Generalization to Other Data Types

Sampling Methods

- The Procedure
  - Sample a subset of the transactions
  - Apply mining algorithm to sampled data
- Challenges
  - False positives: These are patterns that meet the support threshold on the sample but not on the base data.
    - ✓ Post-processing
  - False negatives: These are patterns that do not meet the support threshold on the sample, but meet the threshold on the data.
    - ✓ Reduce the support threshold

Data Partitioned Ensembles

- **The Procedure**
  - The transaction database is partitioned into k disjoint segments
  - The mining algorithm is independently applied to each of these k segments
  - Post-processing to remove false positives
- **Property**
  - No false negatives

Generalization to Other Data Types

- Quantitative Data
  - Rules contain quantitative attributes

$$(Age = 90) \Rightarrow Checkers. \quad Age[85, 95] \Rightarrow Checkers.$$

  - Discretize and converte to binary form
- Categorical Data
  - Rules contain mixed attributes

$$(Gender = Male), \quad Age[20, 30] \Rightarrow Basketball.$$

  - Transform to binary values

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Summary

- Frequent Pattern Mining
  - Support, Downward Closure Property
- Association Rule
  - Support, Confidence
- Frequent Itemset Mining Algorithms
  - Brute Force Algorithms, Apriori, Enumeration-Tree Algorithms, Recursive Suffix-Based Pattern Growth Methods
- Alternative Models: Interesting Patterns
  - Pearson coefficient, χ^2 Measure, Interest Ratio, Symmetric Confidence Measures, …
- Useful Meta-algorithms
  - Sampling, Data Partitioned Ensembles, Generalization